CONDITIONS ON THE SURFACE OF A STRONG DISCONTINUITY IN MULTICOMPONENT MIXTURES

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The differential equations of motion of a multicomponent compressible viscous heat-conducting mixture have been derived under the assumption that the hydrodynamic and physical-chemical characteristics in the stream are continuously differentiable [1-4].

In this paper we shall derive relationships for multicomponent viscous heat-conducting mixtures when the effects of thermal diffusion and pressure-diffusion are taken into account. We shall also obtain relationships on surfaces of discontinuities (of density, pressure, particle velocity, temperature, constituents) to the approximation of boundary-layer theory. For the case of a homogeneous fluid the relationships for strong discontinuities were obtained in [5], likewise to the approximation of boundary-layer theory. As examples, we shall treat the flow past a plane plate when leakage through its surface takes place and the evaporation of the fluid film is taken into account, and the flow past a sublimating wall in an equilibrium gas flow.

1. Derivation of conditions on surfaces of strong discontinuity in multicomponent viscous heat-conducting mixtures. When deriving the conditions on a surface of discontinuity we shall start from the equations of motion of a multicomponent mixture written in integral form.

1. The equation of conservation of mass of the ith component of a mixture is

$$\frac{d}{dt}\int_{V_i^*}\rho_i\,d\tau=\frac{dM_i}{dt}\qquad(i=1,\ldots,N)\tag{1.1}$$

where V_i^* is a certain fluid volume moving with the velocity field $\mathbf{v}_i$ ($\mathbf{v}_i$ is to be taken as the statistically averaged value of the particle velocity of the *i*th component of a mixture relative to some fixed coordinate system), $\rho = n_i m_i$ is the density of the *i*th component, n_i is the number of moles in a unit volume, m_i is the molar mass of the *i*th component, N is the number of components in the system. On the right-hand side of Equation (1.1) is a term which is due to the change of the mass of the *i*th component as the result of chemical reactions.

2. When we sum Equations (1.1) over index i and make use of the law of mass conservation in the chemical reactions, then we obtain the equation of conservation of mass of the mixture

$$\frac{d}{dt}\int\limits_{V^*}\rho\,d\tau = 0 \tag{1.2}$$

where V^* is the volume of a fluid which coincides at the moment under consideration with the volumes V_i^*, but moves with the velocity field

$$\mathbf{v} = \sum_{k=1}^{N} c_k\mathbf{v}_k \qquad \left(c_i = \frac{\rho_i}{\rho},\ \rho = \sum_{k=1}^{N}\rho_k\right)$$

where c_i is the mass concentration of the *i*th component.

3. The equation of conservation of momentum is

$$\frac{d}{dt}\int\limits_{V^*}\rho\mathbf{v}\,d\tau = \int\limits_{S}\mathbf{p}_n\,d\sigma \tag{1.3}$$

where $\mathbf{p}_n$ is the stress tensor on the plane surface, n is the normal to the plane, S is the surface bounding the moving volume V^*. Equation (1.3) is written on the assumption that external body forces do not exist.

4. The equation of conservation of energy is

$$\frac{d}{dt}\int\limits_{V^*}\rho\left(e+\frac{v^2}{2}\right)d\tau = \int\limits_{S}\mathbf{p}_n\mathbf{v}\,d\sigma - \int\limits_{S}\mathbf{J}_q\mathbf{n}\,d\sigma \quad \left(e = \sum_{k=1}^{N} c_\kappa e_\kappa\right) \tag{1.4}$$

where e is the internal energy of a unit mass of mixture, e_i is the partial internal energy of the *i*th component, $\mathbf{J}_q$ is the thermal-energy flux-intensity vector.

If the functions under the integrals are continuous, then from (1.1) to (1.4) we obtain the differential equations of motion of a multicomponent mixture. Let Σ be the isolated piecewise smooth surface of the discontinuity, moving with the normal velocity D and located entirely inside the volume V, which coincides at the moment under consideration with the volume V^*, but moves together with the surface Σ. Then, for any integrable function $A(x, y, z, t)$, the following equation is valid:

$$\frac{d}{dt}\int\limits_V A\,d\tau = \frac{d}{dt}\int\limits_{V^*} A\,d\tau + \int\limits_S A\,(D - v_n)\,d\sigma \qquad (1.5)$$

Noting that when volume V contracts to the surface Σ the left-hand side of Equation (1.5) approaches zero uniformly [6] with respect to time t, we obtain, using Equation (1.5) in Equations (1.1) to (1.4)

$$\rho_1 c_{i1}(D - v_{in,1}) = \rho_2 c_{i2}(D - v_{in,2}) \qquad (i = 1, \ldots, N) \qquad (1.6)$$

$$\rho_1 (D - v_{n1}) = \rho_2 (D - v_{n2}) \qquad (1.7)$$

$$\rho_1 (D - v_{n1})\mathbf{v}_1 + \mathbf{p}_{n1} = \rho_2 (D - v_{n2})\mathbf{v}_2 + \mathbf{p}_{n2} \qquad (1.8)$$

$$\rho_1 (D - v_{n1})\left(e_1 + \tfrac{1}{2}v_1^2\right) + (\mathbf{p}_{n1}\mathbf{v}_1) - (\mathbf{J}_q\mathbf{n})_1 = \rho_2 (D - v_{n2})\left(e_2 + \tfrac{1}{2}v_2^2\right) + (\mathbf{p}_{n2}\mathbf{v}_2) - (\mathbf{J}_q\mathbf{n})_2 \qquad (1.9)$$

If we introduce into Equation (1.6) the densities of the mass streams

$$J_i = \rho_i(\mathbf{v}_i - \mathbf{v}) = \rho_i\mathbf{V}_i, \qquad \mathbf{V}_i = \mathbf{v}_i - \mathbf{v}$$

then Equation (1.6) may be written in the form

$$\rho_1 c_{i1}(D - v_{n1}) - \mathbf{J}_{in,1} = \rho_2 c_{i2}(D - v_{n2}) - \mathbf{J}_{in,2} \qquad (i = 1, \ldots, N) \qquad (1.10)$$

Relation (1.7) is the result of (1.10); note that in the derivation no assumption was made as to the properties of any particular medium.

In distinction to the relations for strong discontinuities on a homogeneous medium [6] the relations (1.7) to (1.10) contain $N - 1$ additional equations, expressing the law of conservation of mass of each component. Furthermore, the heat flux vector here depends not only on the temperature gradient, but also, generally speaking, on the local concentration and pressure gradients [3,4]. Expressions for the vectors $\mathbf{J}_i$ $(i = 1, \ldots, N)$ and $\mathbf{J}_q$ in terms of the macroscopic quantities and physico-chemical parameters of the mixture may be obtained either from the thermodynamics of irreversible processes [3,4] for arbitrary mixtures, or from the kinetic theory of inhomogeneous gases in a non-equilibrium state. Both derivations yield the same results:

$$J_i = \frac{n^2}{\rho}\sum_{\substack{k=1\\k\neq i}}^{N} m_i m_k D_{ik}\left[\nabla c_i^* + c_i(\rho v_i - 1)\nabla \ln p\right] - D_i^T \nabla \ln T \qquad (i = 1, \ldots, N) \qquad (1.11)$$

$$\mathbf{J}_q = -\lambda\nabla T + \frac{p}{n^2}\sum_{k=1}^{N}\sum_{i=1}^{N}\frac{n_i D_k^T}{m_k\mathscr{D}_{ki}}(\mathbf{V}_k-\mathbf{V}_i) + \sum_{k=1}^{N} h_k\mathbf{J}_k \tag{1.12}$$

$$\left(n=\sum_{k=1}^{N} n_k,\ c_i^* = \frac{n_i}{n} = \frac{m}{m_i}c_i,\ m=\frac{\rho}{n}=\sum_{k=1}^{N} c_k^* m_k = \left(\sum_{k=1}^{N}\frac{c_k}{m_k}\right)^{-1},\ \sum_{k=1}^{N} D_k^T = 0\right)$$

where c_i^* is the molar concentration; m is the average molar mass of a mixture; D_{ik} are the multicomponent coefficients of diffusion, which may be expressed according to kinetic theory in the first approximation in terms of $1/2\ N(N-1)$ binary diffusion coefficients $\mathscr{D}_{ik}$ of the various component pairs and the composition of the mixture; λ is the coefficient of heat conductivity of the mixture; D_i^T are the multicomponent thermal-diffusion coefficients; h_i and v_i are the partial enthalpy and the volume of the ith component, respectively.

In the case of mixtures of fluids (solutions, mixtures of gases) the expression for the stress-tensor $\mathbf{p}_n$ in (1.8) is identical with the expression obtained in the hydrodynamics of a homogeneous fluid if the influence of chemical reactions on the stress tensor is not taken into account. The coefficients D_{ik}, D_i^T, λ and μ depend on pressure, temperature and the composition of the mixture, and will be referred to as known functions of the characteristics of the mixture.

In the case of a binary ideal mixture containing components i and j, Expressions (1.11) and (1.12) assume a particularly simple form

$$\mathbf{J}_i = -\mathbf{J}_j = -\rho\mathscr{D}_{ij}\left(\nabla c_i + \alpha_p c_i c_j\nabla\ln p + \frac{m_i m_j}{m^2}\alpha_T c_i c_j\nabla\ln T\right) \tag{1.13}$$

$$\mathbf{J}_q = -\lambda\nabla T + \alpha_T\frac{p}{\rho}\mathbf{J}_i + (h_i-h_j)\mathbf{J}_i \quad \alpha_p = (m_j-m_i)\left(\frac{c_i}{m_i}+\frac{c_j}{m_j}\right), \tag{1.14}$$

$$c_i + c_j = 1$$

where α_T is the thermal-diffusion constant.

If no limitations are imposed upon the order of magnitude of the gradients of the macroscopic physical quantities (temperature, composition), then when the Reynolds number approaches infinity, the relationships (1.7) to (1.10) are converted into the relationships for shock waves in an ideal (nonviscous non-heat-conducting) mixture composed of N components

$$\rho_1 c_{i1}(D-v_{n1}) = \rho_2 c_{i2}(D-v_{n2}) \qquad (i=1,\ldots,N) \tag{1.15}$$

$$\rho_1(D-v_{n1}) = \rho_2(D-v_{n2}) \tag{1.16}$$

$$\rho_1(D-v_{n1})\mathbf{v}_1 - p_1\mathbf{n} = \rho_2(D-v_{n2})\mathbf{v}_2 - p_2\mathbf{n} \tag{1.17}$$

$$\rho_1(D-v_{n1})\left(e_1+\tfrac{1}{2}v_1^2\right) - p_1 v_{n1} = \rho_2(D-v_{n2})\left(e_2+\tfrac{1}{2}v_2^2\right) - p_2 v_{n2} \tag{1.18}$$

Let us derive the relations on a surface of strong discontinuity to the approximation of the boundary-layer theory ($R \to \infty$).

Let $S(x, y, z) = 0$ be the equation of the surface past which the flow takes place. We shall assume the function S to be an analytical regular function, such that at any point the surface $S = 0$ has a non-zero radius of curvature. If we choose the traces of curvature to be the coordinate system x, z on the surface $S = 0$, then the system of surfaces parallel to the surface $S = 0$ and the family of unfolding surfaces, formed by the normals to the surface $S = 0$ along the coordinate lines x, z, form a triply orthogonal system of surfaces [7]. The linear coordinate along the normal will be denoted by y. In this triply orthogonal curvilinear coordinate system we obtain relations (1.7) to (1.10) for $R \sim 0(l/\delta)^2$, where l is a characteristic length, δ is the thickness of the boundary layer. Omitting the derivations, the results are

$$[\rho c_i (D + u \tan\beta + w \tan\gamma - v) - J_{iy}] = O\left(\frac{1}{\sqrt{R}}\right) \qquad (i = 1, \ldots, N) \tag{1.19}$$

$$[\rho (D + u \tan\beta + w \tan\gamma - v)] = O\left(\frac{1}{\sqrt{R}}\right) \tag{1.20}$$

$$[p] = O\left(\frac{1}{R}\right) \tag{1.21}$$

$$\left[\rho (D + u \tan\beta + w \tan\gamma - v)\, u + \mu \frac{\partial u}{\partial y}\right] = O\left(\frac{1}{R}\right) \tag{1.22}$$

$$\left[\rho (D + u \tan\beta + w \tan\gamma - v)\, w + \mu \frac{\partial w}{\partial y}\right] = O\left(\frac{1}{R}\right) \tag{1.23}$$

$$\Big[\rho (D + u \tan\beta + w \tan\gamma - v)\left(e + \frac{u^2 + w^2}{2}\right) + p\,(u \tan\beta + w \tan\gamma - v) + \\ + \mu \frac{\partial}{\partial y}\left(\frac{u^2 + w^2}{2}\right) - J_{qv}\Big] = O\left(\frac{1}{R}\right) \tag{1.24}$$

or, if we introduce the enthalpy of the mixture $h = e + p/\rho$

$$\left[\rho (D + u \tan\beta + w \tan\gamma - v)\left(h + \frac{u^2 + w^2}{2}\right) + \mu \frac{\partial}{\partial y}\left(\frac{u^2 + w^2}{2}\right) - J_{qv}\right] = O\left(\frac{1}{R}\right) \tag{1.25}$$

Here and in what follows, the square brackets will represent the jump of the quantities therein: β is the angle in the plane $z = \text{const}$ between the tangent to the discontinuity surface at a given point and the tangent to the contour of the body at the point of the same x; γ is the angle in the plane $x = \text{const}$ between the tangent to the discontinuity surface at a given point and the tangent to the surface of the contour at the same z

$$J_{iv} = \frac{n^2}{\rho} \sum_{\substack{k=1 \\ k \neq i}}^{N} m_i m_k D_{ik} \frac{\partial c_i^{\,*}}{\partial y} - D_i^{T} \frac{\partial \ln T}{\partial y} \tag{1.26}$$

$$J_{qy} = -\lambda \frac{\partial T}{\partial y} + \frac{p}{n^2} \sum_{k=1}^{N} \sum_{i=1}^{N} \frac{n_i D_k{}^T}{m_k \mathscr{D}_{ki}} (V_{ky} - V_{iy}) + \sum_{k=1}^{N} h_k J_{ky} \qquad (1.27)$$

Note that to the approximation of boundary-layer theory the effect of pressure diffusion drops out of Expressions (1.26) and (1.27). In the case of a binary ideal mixture the expressions for J_{iy} and J_{qy} in (1.19), (1.24) and (1.25) assume a particularly simple form

$$J_{iy} = -J_{jy} = -\rho \mathscr{D}_{ij} \left(\frac{\partial c_i}{\partial y} + \frac{m_i m_j}{m^2} \alpha_T c_i c_j \frac{\partial \ln T}{\partial y} \right) \qquad (1.28)$$

$$J_{qy} = -\lambda \frac{\partial T}{\partial y} + \alpha_T \frac{p}{\rho} J_{jy} + (h_i - h_j) J_{iy} \qquad (1.29)$$

In the case of a homogeneous fluid the relationships (1.19) to (1.25) become the known relationships for plane parallel motion [5].

In the case of an inhomogeneous viscous heat-conducting fluid, relations (1.19) to (1.25) alone are not sufficient for a single-valued transition through the surface of discontinuity. This was noted to be the case for a homogeneous fluid in [5]. For a single-valued transition through the surface of discontinuity it is necessary to impose additional conditions.

In the case of a sufficiently dense viscous heat-conducting mixture it is natural to assume equality of the tangential components of the particle velocity at the discontinuity surface, and equality of the temperatures on both sides of the surface of discontinuity. For $R \to \infty$ these conditions assume the form

$$u_1 = u_2, \qquad w_1 = w_2, \qquad T_1 = T_2 \qquad (1.30)$$

The additional relationships for the concentration of gas components on the discontinuity surface may be different and may be determined from the physical nature of that surface. From the theoretical investigations [8] and many experimental results (see, for example, [9]), it follows that the process of evaporation of gases in a stream takes place according to the kinetics of diffusion, i.e. the partial pressure of the gases on the surface of evaporation is equal to the saturation pressure for the temperature of the surface. Consequently an additional relation in the case of evaporation will connect the pressure to the temperature

$$p_i = f(T) \qquad (1.31)$$

where p_i is the partial pressure of the gas on the evaporating surface and T is the temperature of the surface. If we approximate and assume that the latent heat of evaporation l does not depend on temperature, then [10]

$$p_i = \exp\left(-\frac{l}{RT} + a\right) \tag{1.32}$$

where a is a constant for a given substance. In place of (1.31) it is convenient to have a relation between the mass concentration c_i and T on the surface of evaporation. Using the equality $c_i^* = p_i/p$, we obtain

$$c_i = m_i\left[\frac{p}{f(T)} - 1\right]^{-1}\sum_{k\neq i}^{N}\frac{c_k}{m_k} \tag{1.33}$$

where p is the static pressure of the mixture on the surface of evaporation (in the approximation of boundary-layer theory $\partial p/\partial y = 0$ and $p = p_\infty$ is the pressure outside the boundary layer).

Under certain conditions, which may be determined only after the evaporation problem is solved, the pressure of the gases in some regions of the surface of evaporation may reach the outside pressure p_∞. Then boiling will begin in these regions and an additional condition on the surface of discontinuity in these regions well be

$$p_i = p_\infty \tag{1.34}$$

Since the external pressure varies along the surface of discontinuity, then because of (1.31) and (1.34) the temperature will also vary along the boiling surface.

In the case of sublimation of certain substances, for example, graphite, intensive evaporation begins at a certain temperature T_* characteristic of a given substance. At a surface temperature smaller than T_* the evaporation may be neglected. In this case, in addition to (1.30), we may establish the condition $T_1 = T_2 = T_*$. The composition on the surface of sublimation in this case is determined during the process of solution.

In the case of strong variations of the outside pressure along the body contour at the points of the body where the pressure is greater than the pressure of the triple point of the phase diagram, but smaller than the pressure at the critical point, the body, for sufficiently great heat flows, will melt on the side to the gas, accompanied by evaporation (or even by boiling) of the fluid film; in the case of small heat flows it will sublimate. At the points where the pressure is smaller than the pressure at the triple point, the body will sublimate. Consequently, in the general case, lines will form on the surface of the body dividing the regions of melting with evaporation and the regions of sublimation. The location of these lines is not known beforehand and must be found in the process of solution. Because of the assumptions (1.30), relations (1.22) to (1.25) are significantly simplified:

$$\left[\mu \frac{\partial u}{\partial y}\right] = O\left(\frac{1}{\sqrt{R}}\right), \qquad \left[\mu \frac{\partial w}{\partial y}\right] = O\left(\frac{1}{\sqrt{R}}\right) \tag{1.35}$$

$$[\rho (D + u \tan \beta + w \tan \gamma - v) e - pv - J_{qy}] = O\left(\frac{1}{\sqrt{R}}\right) \tag{1.36}$$

or

$$[\rho (D + u \tan \beta + w \tan \gamma - v) h - J_{qy}] = O\left(\frac{1}{\sqrt{R}}\right) \tag{1.37}$$

As an example we shall write down the set of final conditions, which determine the single-valued transition through the surface of evaporation from the fluid phase past which a mixture of gases flow. Combining conditions (1.19), (1.20) and (1.37), we obtain

$$\rho (D + u \tan \beta + w \tan \gamma - v)(1 - c_i) + J_{iy} = 0$$

$$\rho c_k (D + u \tan \beta + w \tan \gamma - v) - J_{ky} = 0 \qquad (k = 1, \ldots, i-1, i+1, \ldots, N)$$

$$\rho (D + u \tan \beta + w \tan \gamma - v) = \rho_1 (D + u_1 \tan \beta + w_1 \tan \gamma - v_1)$$

$$p = p_1, \qquad \mu \frac{\partial u}{\partial y} = \mu_1 \frac{\partial u_1}{\partial y}, \qquad \mu \frac{\partial w}{\partial y} = \mu_1 \frac{\partial w_1}{\partial y} \tag{1.38}$$

$$\rho (D + u \tan \beta + w \tan \gamma - v) l - \frac{p}{n^2} \sum_{k=1}^{N} \sum_{l=1}^{N} \frac{n_l D_k{}^T}{m_k \mathscr{D}_{kl}} (V_{ky} - V_{ly}) = \lambda_1 \frac{\partial T_1}{\partial y} - \lambda \frac{\partial T}{\partial y}$$

$$u = u_1, \qquad w = w_1, \qquad T = T_1, \qquad c_i = m_i \left(\frac{p}{f(T)} - 1\right)^{-1} \sum_{k \neq i}^{N} \frac{c_k}{m_k}$$

where quantities without indices refer to the gas mixture on the surface of evaporation, the quantities with index 1 refer to the liquid phase on the surface of evaporation and those with index i to the vapor components. Direct counting of conditions (1.38) shows that they are sufficient for a single-valued transition through the surface of evaporation.

The compatibility conditions on the surface of combustion may also be obtained from conditions (1.7) to (1.10) in multiple-component mixtures by adding additional assumptions about the kinetics at the front of combustion.

2. Example 1. Let us consider the flow past a flat plate with diffusion of a fluid through its surface when taking into account the evaporation of the film.

Let it be assumed that a fluid diffuses through the surface of a plate, past which flows a uniform plane parallel stream of gas. This fluid forms a thin film along the plate which is dragged by the outside stream. If the outside stream is a hot gas, then the liquid film will evaporate intensively, thereby diminishing the heat flow directed to the wall (shielded cooling). The solution of this problem is reduced to the solution of a system of equations of the boundary-layer type for a binary mixture gas-vapor in the outside stream

$$\begin{gathered}
\frac{\partial}{\partial x}(\rho_1 u_1)+\frac{\partial}{\partial y}(\rho_1 v_1)=0, \qquad \rho_1\left(u_1\frac{\partial u_1}{\partial x}+v_1\frac{\partial u_1}{\partial y}\right)=\frac{\partial}{\partial y}\left(\mu_1\frac{\partial u_1}{\partial y}\right)\\
\rho_1\left(u_1\frac{\partial c}{\partial x}+v_1\frac{\partial c}{\partial y}\right)=\frac{\partial}{\partial y}\left(\rho_1\mathscr{D}_{12}\frac{\partial c}{\partial y}\right) \qquad (2.1)\\
\rho_1\left(u_1\frac{\partial H_1}{\partial x}+v_1\frac{\partial H_1}{\partial y}\right)=\frac{\partial}{\partial y}\left\{\frac{\mu}{P}\left[\frac{\partial H_1}{\partial y}+(P_1-1)\frac{\partial}{\partial y}\left(\frac{u_1^2}{2}\right)+\left(\frac{1}{L}-1\right)\left(h^{(1)}-h^{(2)}\frac{\partial c}{\partial y}\right)\right]\right\}\\
H_1=h_1+\frac{u_1^2}{2}, \qquad p_\infty=R\rho_1 T_1\left(\frac{c}{m_1}+\frac{1-c}{m_2}\right)\\
\left(P_1=\frac{\mu c_p}{\lambda}-\text{Prandtl number}, L=\frac{\lambda}{\rho c_p D_{12}}-\text{Lewis number}\right)
\end{gathered}$$

and the boundary-layer equations for a liquid film

$$\begin{gathered}
\frac{\partial}{\partial x}(\rho_2 u_2)+\frac{\partial}{\partial y}(\rho_2 v_2)=0, \qquad \rho_2\left(u_2\frac{\partial u_2}{\partial x}+v_2\frac{\partial u_2}{\partial y}\right)=\frac{\partial}{\partial y}\left(\mu_2\frac{\partial u_2}{\partial y}\right), \qquad p_2=p_2(\rho_2,T_2) \qquad (2.2)\\
\rho_2\left(u_2\frac{\partial H_2}{\partial x}+v_2\frac{\partial H_2}{\partial y}\right)=\frac{\partial}{\partial y}\left\{\frac{\mu_2}{P_2}\left[\frac{\partial H_2}{\partial y}+(P_2-1)\frac{\partial}{\partial y}\left(\frac{u^2}{2}\right)\right]\right\}, H_2=h_2+\frac{u_2^2}{2}
\end{gathered}$$

These systems of equations may be solved using the following boundary conditions:

1) in the stream at infinity

$$u_1=u_\infty, \qquad c=c_\infty, \qquad h_1=h_\infty \qquad (2.3)$$

2) on the surface of evaporation ($D=0$)

$$\begin{gathered}
\rho_1(u_1\tan\beta-v_1)(1-c_0)-\rho_1\mathscr{D}_{12}\frac{\partial c}{\partial y}=0, \qquad \mu_1\frac{\partial u_1}{\partial y}=\mu_2\frac{\partial u_2}{\partial y}\\
\rho_1(u_1\tan\beta-v_1)=\rho_2(u_2\tan\beta-v_2), \qquad u=u_1, \qquad T_1=T_2=T_0 \qquad (2.4)\\
\rho_1 l(T_0)(u_1\tan\beta-v_1)=\lambda_2\frac{\partial T_2}{\partial y}-\lambda_1\frac{\partial T_1}{\partial y}, \ c_0=\left[1+\left(\frac{p_\infty}{f(T_0)}-1\right)\frac{m_2}{m_1}\right]^{-1}
\end{gathered}$$

3) on the porous plate

$$u_2=0, \qquad v_2=\frac{\rho_\infty}{2\rho_2}\left(\frac{\nu_\infty u_\infty}{x}\right)^{1/2}B, \qquad H_2=h_w \qquad (2.5)$$

where the index ∞ refers to the values of the parameters outside the boundary layer, the index 1 to the binary stream in the boundary layer, the index 2 to the film, the index 0 to the still unknown values of concentration and temperature on the surface of evaporation, the index w to the values of the quantities on the wall; u and v are the components of the velocity along the x- and y-axes, respectively, along the plate and its normal, c is the concentration of the vapor, $h^{(1)}$ and $h^{(2)}$ are the partial specific enthalpies of vapor and gas respectively, m_1 and m_2 are the molar masses of vapor and gas, respectively, B is a given dimensionless constant connected with the mass dispersion of the liquid Q_c through the surface of a plate of length l given by the formula

$$B = \frac{Q_c}{\rho_\infty u_\infty l}\sqrt{R}, \qquad R = \frac{u_\infty l \rho_\infty}{\mu_\infty} \tag{2.6}$$

The following assumptions were made in formulating this problem: the motion of the film and of the binary mixture under the film is laminar; there are no external mass forces; the effect of thermal diffusion is small and may be neglected in comparison with the ordinary diffusion in the diffusion equation and in comparison with the heat conductivity in the energy equation. The feeding of the liquid through the plate takes place according to the special law

$$v_2(x, 0) \sim x^{-1/2}$$

If, further, we make the assumption that $P_1 = P_2 = L = 1$, then it is easy to note that the equation of diffusion and the equation of energy allow the following integrals:

$$c = \alpha + \beta u_1, \quad h_1 \equiv ch^{(1)} + (1 - c)\,h^{(2)} = \alpha_1 + \beta_1 u_1 - \tfrac{1}{2}u_1^2, \quad h_2 = \alpha_2 + \beta_2 u_2 - \tfrac{1}{2}u_2^2 \tag{2.7}$$

where α and β are constants of integration. The first two integrals are direct generalizations of the Crocco integral for the case of a binary stream.

Investigating this case further, to simplify the calculation we shall look for a solution of the system of equations (2.1) and (2.2) in the form

$$\rho_i u_i = \varphi_i'(\eta), \qquad \rho_i v_i = {}^1\!/_2\left(\frac{\nu_\infty}{u_\infty x}\right)^{1/2}[\eta\varphi_i'(\eta) - \varphi_i(\eta)], \qquad h_i = \alpha_i + \beta_i u_i - \frac{u_i^2}{2} \quad (i = 1, 2)$$

$$c = \alpha + \beta u_1(\eta), \qquad \eta = \left(\frac{u_\infty}{\nu_\infty x}\right)^{1/2} y \tag{2.8}$$

where the density and the velocity components u and v are made dimensionless with respect to ρ_∞ and u_∞, respectively, and the enthalpies h_i ($i = 1, 2$) with respect to u_∞^2. Substituting Expressions (2.8) into Equations (2.1), (2.2) and boundary conditions (2.3) to (2.5), we arrive at the following boundary-value problem:

$$\frac{d}{d\eta}\left(\mu_i \frac{du_i}{d\eta}\right) + \frac{\varphi_i}{2}\frac{du_i}{d\eta} = 0, \qquad \rho_i u_i = \varphi_i'(\eta) \qquad (i = 1, 2) \tag{2.9}$$

$$u_1(\infty) = 1, \qquad \varphi_1(\eta^*) = \varphi_2(\eta^*), \qquad u_1 = u_2 = u^*, \qquad \tan\beta = \frac{1}{2}\left(\frac{\nu_\infty}{u_\infty x}\right)^{1/2}\eta^*$$

$$\left(\mu\frac{\partial u}{\partial\eta}\right)_1 = \left(\mu\frac{\partial u}{\partial\eta}\right)_2, \qquad u_2(0) = 0, \qquad \varphi_2(0) = -B$$

$$\frac{\varphi_1(\eta^*)}{2}(1 - c_0)(1 - u^*) = \left(\mu\frac{\partial u}{\partial\eta}\right)_1 (c_\infty - c_0), \qquad c_0 = \psi(T_0) \tag{2.10}$$

$$l(T_0)\frac{\varphi_1(\eta^*)}{2} = \left(\mu\frac{\partial u}{\partial\eta}\right)_1 \left[\frac{h_{20} - h_w - {}^1/_2 u^{*2}}{u^*} - \frac{h_\infty - h_0^{(2)} + {}^1/_2(1-u^*)^2 + c_\infty(h_0^{(2)} - h_0^{(1)})}{1 - u^*}\right]$$

where the latent heat of evaporation $l(T_0)$ is measured in units of u_∞^2, μ in the units of μ_∞, $c_0 = \psi(T_0)$ is the short notation of the vapor pressure curve.

Every equation of (2.9) is of the third order (because $\rho_i u_i = \phi_i'$) and, furthermore, η^*, u^*, c_0 and T_0 are four unknowns. Relationships (2.10) give us exactly ten conditions. After this boundary problem is solved, the fields of concentration and of enthalpy are determined from the equations

$$c = \frac{c_0 - c_\infty u^*}{1 - u^*} + \frac{c_\infty - c_0}{1 - u^*}u \tag{2.11}$$

$$h_1 = \frac{h_{10} - u^* h_\infty - {}^1/_2 u^*(1 - u^*)}{1 - u^*} + \frac{h_\infty - h_{10} + {}^1/_2(1 - u^{*2})}{1 - u^*}u_1 - \frac{u_1^2}{2} \tag{2.12}$$

$$h_2 = h_w + \frac{h_{20} - h_w + {}^1/_2 u^{*2}}{u^*}u_2 - \frac{u_2^2}{2} \tag{2.13}$$

To solve the problem (2.9) to (2.10) we shall introduce a new dependent variable $\omega = \mu\partial u/\partial\eta$, and we choose u to be the independent variable [11, 12]. Then, in terms of these new variables, we have the following boundary-value problem:

$$\frac{d^2\omega_i}{du_i^2} + \rho_i\mu_i\frac{u_i}{2\omega_i} = 0 \qquad (i = 1, 2) \tag{2.14}$$

$$\omega_1(1) = 0, \qquad \omega_1(u^*) = \omega_2(u^*), \qquad \omega_2'(u^*) = \omega_2'(u^*), \qquad \omega_2'(0) = {}^1/_2 B$$

$$\omega_1'(u^*)(1 = c_0)(1 - u^*) = \omega_1(u^*)(c_0 - c_\infty), \qquad c_0 = \psi(T_0)$$

$$\omega_1'(u^*)\, l(T_0) = \omega_1(u^*)\, A(T_0, u^*) \tag{2.15}$$

where

$$A(T_0, u^*) = \frac{h_\infty - h_0^{(2)} + {}^1/_2(1 - u^{*2}) + c_\infty(h_0^{(2)} - h_0^{(1)})}{1 - u^*} - \frac{h_{20} - h_w - {}^1/_2 u^{*2}}{u^*}$$

The system of two equations of the second order (2.14) with three unknown parameters u^*, c_0 and T_0 with the boundary conditions (2.15) has to be solved with the help of seven conditions (2.15). Consequently it is expected that a unique solution of the problem will be obtained. To simplify the solution we shall assume that the quantity $\rho_i\mu_i$ is constant in every region. Without restricting the generality of the solution we assume

$$\rho_1\mu_1 = 1, \qquad \rho_2\mu_2 = K^2$$

We shall look for the solution of the problem in the form

$$\omega_1 = \omega^\circ_C(u), \qquad \omega_2 = K\alpha\omega_{\gamma_2}(\alpha^{-2/3}u)$$

where $\omega_C^\circ(u)$ is the single-parameter family of solutions of equation $2\omega\omega'' + u = 0$, determined for $u < 1$ and satisfying the condition $\omega_C^\circ(1) = 0$, $\omega_{\gamma 2}(u)$ is the solution of a Cauchy problem:

$$2\omega\omega'' + u = 0, \qquad \omega_{\gamma_2}(0) = 1, \qquad \omega_{\gamma_2}'(0) = \tan\gamma_2$$

(both families of the solutions ω_C° and $\omega_{\gamma 2}$ have been studied in [5]). where α is an additional parameter subject to further determination. With function ω_1 so chosen, the boundary condition $\omega_1(1) = 0$ is satisfied automatically. The remaining conditions (2.15) give a system of six transcendental equations

$$\omega_C^\circ(u^*) = K\alpha\omega_{\gamma_2}(\alpha^{-2/3}u^*), \qquad \omega_C^{\circ\prime}(u^*) = K\alpha^{1/3}\omega_{\gamma_2}'(\alpha^{-2/3}u^*), \; K\alpha^{1/3}\tan\gamma_2 = \frac{1}{2}B \quad (2.16)$$

$$\frac{\omega_C^{\circ\prime}(u^*)}{\omega_C^\circ(u^*)} = \frac{c_0 - c_\infty}{(1 - c_0)(1 - u^*)}, \qquad \frac{\omega_C^{\circ\prime}(u^*)}{\omega_C^\circ(u^*)} = \frac{A(T_0, u^*)}{l(T_0)}, \qquad c_0 = \psi(T_0)$$

for the determination of six unknown quantities: C, u^*, α, γ_2, c_0 and T_0. The functions $\omega_C^\circ(u)$ which do not vanish between 0 and 1, are given by the formula

$$\omega_C^\circ(u) = u_0^{-3/2}\omega_{\gamma_1}(uu_0), \qquad \omega_C^{\circ\prime}(u) = u_0^{-1/2}\omega_{\gamma_1}'(uu_0)$$

where $u_0(\gamma_1)$ are the zeros of the functions $\omega_{\gamma 1}$, (u). Therefore, for these functions the angle $\gamma_1 (C = \gamma_1)$ and

$$\frac{\omega_C^{\circ\prime}(u^*)}{\omega_C^\circ(u^*)} = u_0\frac{\omega_{\gamma_1}'(u^*u_0)}{\omega_{\gamma_1}(u^*u_0)}$$

serve as parameters instead of C.

The values of the functions $\omega_{\gamma 1}(u)$ and $u_0(\gamma_1)$ are adopted from [5] in the form of a table, which is supplemented by the values of the derivatives $\omega_{\gamma 1}'(u)$.

For large values of $\tan\gamma_1$ the asymptotic representation $\omega_{\gamma 1}(u)$ may be used [5]. Then we obtain

$$\omega_C{}^\circ(u) = X^{-3/2}\omega_0(uX), \qquad \omega_C{}^{\circ\prime}(u) = X^{-1/2}\omega_0{}'(uX), \qquad X = 2.608 \qquad (2.17)$$

where $\omega_0(u)$ is the solution of the Cauchy problem

$$2\omega\omega'' + u = 0, \qquad \omega(0) = 0, \qquad \omega'(0) = 1$$

The corresponding relation

$$\frac{\omega_C{}^{\circ\prime}(u^*)}{\omega_C{}^\circ(u^*)} = X\frac{\omega_0{}'(u^*X)}{\omega_0(u^*X)}$$

for large tan γ_1 does not depend on the parameter γ_1. Functions $\omega_C^\circ(u)$, having another zero in (0, 1), are equal

$$\omega_C{}^\circ(u) = \omega_\xi{}^\circ(u) \qquad (C = \xi)$$

where the functions ω_ξ° are the solutions of the boundary-value problem

$$2\omega\omega'' + u = 0, \qquad \omega(\xi) = 0, \qquad \omega(1) = 0, \qquad (0 \leqslant \xi \leqslant 1)$$

and tables of these functions are given in [5]. Since for $\gamma_1 = \pi/2$ and $\xi = 0$ the integral curves coincide, the functions (2.17) may be used in this case in place of the functions $\omega_0{}^\circ(u)$ and $\omega_0{}^{\circ\prime}(u)$.

Direct determination of parameters C, u^*, a, γ_2, c_0 and T_0 from the system (2.16) requires extensive calculations for each group of given parameters. However, if we apply a semi-inverse method of solution to the system (2.16), then system (2.16) may be solved consecutively without difficulty. Indeed, let the quantity u^* be given (the approximate value of the dimensionless velocity u^* on the surface of evaporation may be assumed to be equal to the value u^* disregarding the evaporation from [5]). Then from the last three equations (2.16) we find the values c_0, T_0 and C at once. Knowing u^* and C, we find a and γ_2 from the first two equations of (2.16) using a graphical method. The third relation will give the value of the parameter of inflow B corresponding to a given value u^* and given values of the parameters of the problem. Two or three trials in the choice of u^* lead to the desired value B.

In an analogous way the system (2.16) may be solved by choosing c_0 (the concentration on the surface of evaporation). Upon solution of the system of equations (2.16), from Equations (2.11) to (2.13) the fields of concentrations and enthalpies and the coefficient of friction are found:

$$c_f\sqrt{R_x} = 2\omega_2(0) = 2K\alpha, \qquad R_x = \frac{u_\infty x\rho_\infty}{\mu_\infty}$$

and so is the heat flow q_w toward the plate

$$\frac{q_w \sqrt{R_x}}{\rho_\infty u_\infty} = K\alpha \frac{h_{20} - h_w + {}^1/_2 u^{*2} u_\infty^2}{u^*}$$

where $h_{20} - h_w$ is the dimensional difference of enthalpies of the liquid on the surface of evaporation and on the wall.

3. Example 2. Let us investigate the sublimation of a plane wall in a uniform gas flow. Let a uniform gas stream move along a heat-conducting wall with a velocity u_∞. If the static pressure p_∞ is smaller than the pressure at the triple point of the phase diagram of the wall substance, then, depending on the parameters of the oncoming stream and the conditions of the heat exchange on the other side of the wall, the surface of the wall which is directed toward the gas will sublimate (evaporate, bypassing the liquid phase) or, if the pressure is larger the vapor pressure of the substance of the wall (if such exist) it will condense from the stream immediately into the solid phase. If the x-coordinate is measured along the wall, and the y-coordinate normal to the wall, then this problem will reduce to the solution of the system of equations for the non-steady (because of the change in one boundary of the wall) binary boundary layer and of the equation of heat conductivity in the wall

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \mathrm{v}) = 0, \qquad \rho \frac{du}{dt} = \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right), \qquad \rho \frac{dc}{dt} = \frac{\partial}{\partial y}\left(\rho \mathscr{D}_{12} \frac{\partial c}{\partial y}\right) \tag{3.1}$$

$$\rho \frac{dH}{dt} = \frac{\partial}{\partial y}\left\{\frac{\mu}{P}\left[\frac{\partial H}{\partial y} + (P-1)\frac{\partial}{\partial y}\left(\frac{u^2}{2}\right) + \left(\frac{1}{L} - 1\right)(h^{(1)} - h^{(2)})\frac{\partial c}{\partial y}\right]\right\}$$

$$\rho_1 c_1 \frac{\partial T_1}{\partial t} = \frac{\partial}{\partial y}\left(\lambda_1 \frac{\partial T_1}{\partial y}\right) \tag{3.2}$$

with the following conditions:

1) in the stream of infinity

$$u = u_\infty, \qquad h = h_\infty, \qquad c = c_\infty \tag{3.3}$$

2) on a yet unknown surface of sublimation

$$\rho(D - v)(1 - c_0) = \rho \mathscr{D}_{12} \frac{\partial c}{\partial y}, \qquad \rho(D - v) = \rho_1 D, \qquad u = 0, \qquad T = T_1 = T_0$$

$$\rho(D - v)\, l(T_0) = \lambda_1 \frac{\partial T_1}{\partial y} - \lambda \frac{\partial T}{\partial y}, \qquad c_0 = \psi(T_0) \tag{3.4}$$

3) in the wall at infinity

$$T_1 = T_{-\infty} \tag{3.5}$$

where index 1 refers to the parameters of the wall, index 0 to the parameters on the surface of sublimation, which are known only after the problem is solved. The remaining notation coincides with the notation of

Section 2.

In writing the equation of heat conductivity in a body (3.2), the assumption was made that the thickness of the thermal boundary layer in the body is sufficiently small compared with the characteristic length of the wall. Therefore, in the equation of heat conductivity the derivatives with respect to x may be neglected relative to the derivatives with respect to y.

In addition to conditions (3.3) to (3.5) it is necessary to write the initial conditions of the problem.

If we limit the investigation to the particular case where $P = L = 1$ and look for the temperature of the surface of sublimation, then the solution may be obtained in closed form by making certain assumptions. Let us make the basic assumption: that the velocity of propagation of the surface of sublimation D depends only on the x-coordinate and does not depend on time. Thereby is established at each point of the surface a particular stationary mode of sublimation depending on x. The conditions under which this assumption is fulfilled will be shown below. Then the solution of the equation of energy diffusion and the equation of heat conductivity, taking the boundary conditions (3.3) to (3.5) into account, will be

$$c = c_0 + (c_\infty - c_0)\frac{u(\eta)}{u_\infty}, \qquad h = h_0 + (H_\infty - h_0)\frac{u(\eta)}{u_\infty} - \frac{1}{2}u^2(\eta) \tag{3.6}$$

$$T_1 = T_{-\infty}\theta_1(\eta_1), \quad \eta = \sqrt{\frac{u_\infty}{v_\infty x}}\,(y - Dt), \quad \eta_1 = \frac{D}{\chi_1}(y - Dt), \quad \chi_1 = \frac{\lambda_1(T_{-\infty})}{\rho_1(T_{-\infty})\,c_1(T_{-\infty})}$$

where the function $\theta_1(\eta_1)$ is the solution of the boundary-value problem

$$\frac{d}{d\eta_1}\left[L_1(\theta_1)\frac{d\theta_1}{d\eta_1}\right] + N(\theta_1)\frac{d\theta_1}{d\eta_1} = 0, \qquad \theta_1(0) = \frac{T_0}{T_{-\infty}} = n, \qquad \theta_1(\infty) = 1 \qquad (D < 0) \tag{3.7}$$

which upon integration by two quadratures finally becomes [13]

$$\eta_1 = \int_{\theta_1}^{n} \frac{L(\theta_1)\,d\theta_1}{M(\theta_1) - M(1)}, \qquad \frac{d\theta_1}{d\eta_1} = \frac{M(1) - M(\theta_1)}{L(\theta_1)} \tag{3.8}$$

where

$$\lambda_1(T_1) = \lambda_1(T_{-\infty})\,L(\theta_1), \qquad \rho_1(T_1)\,c_1(T_1) = \rho_1(T_{-\infty})\,c_1(T_{-\infty})\,N(\theta_1)$$

$$M(\theta_1) = \int N(\theta_1 d\theta_1)$$

In the solution (3.6) the quantities with index 0 are unknown and must be determined from conditions (3.4), which yield two transcendental equations

$$\frac{c_0 - c_\infty}{1 - c_0} = \frac{H_\infty - h_0{}^{(2)} + c_\infty (h_0{}^{(2)} - h_0{}^{(1)})}{l\,(T_0) + c_1\,(T_{-\infty})\,T_{-\infty}\,[M\,(n) - M\,(1)]}, \qquad c_0 = \psi\,(T_0) \tag{3.9}$$

for the determination of the concentration c_0 and the temperature T_0 on the surface of sublimation. From the system (3.9) it follows that the temperature of the evaporating surface approaches the temperature of boiling only for $H_\infty \to \infty$, i.e. in the case of infinitely great heat inflow from the gas.

Let us show now under what conditions the solution obtained above is valid. Consider the solution of the problem (3.1) to (3.5) in the following form (the normal velocity of propagation of the surface of sublimation does not depend on time):

$$\rho u = \varphi_\eta'\,(\eta,\, t), \qquad \rho v = \frac{1}{2}\left(\frac{\nu_\infty}{u_\infty x}\right)^{1/2}\left[\eta\varphi_\eta' - \varphi\,(\eta,\, t) + \frac{\rho}{\rho_1}\,\varphi(0,\, t)\right]$$

$$\rho_1 D = \frac{1}{2}\left(\frac{\nu_\infty u_\infty}{x}\right)^{1/2}\varphi\,(0,\, t) \tag{3.10}$$

where velocities u and v and densities ρ and ρ_1 are measured in terms of the quantities u_∞ and ρ_∞. The substitution of Expressions (3.10) into the equation of motion yields

$$\frac{\partial}{\partial\eta}\left(\frac{\mu}{\mu_\infty}\frac{\partial u}{\partial\eta}\right) + \frac{\varphi}{2}\frac{\partial u}{\partial\eta} = \frac{\rho}{2\rho_1}\varphi(0,\tau)\frac{\partial u}{\partial\eta} + \rho x'\frac{\partial u}{\partial\tau} + \frac{\tau}{4x'\rho_1}\varphi\,(0,\,\tau)\frac{\partial\varphi}{\partial\eta} \tag{3.11}$$

where $\tau = u_\infty t/l$. The first term on the right-hand side of this equation may be neglected, since for all condensing media $\rho/\rho_1 \sim 10^{-5} \div 10^{-4}$. The second term on the right-hand side is also small, since the characteristic time l/u_∞ for all except very small velocities is small. This approximation physically corresponds to the obvious fact that the local acceleration produced by the motion of the surface due to sublimation is small compared with the convective acceleration.

The last term in Equation (3.11) is small only for sufficiently small initial intervals of time. However, since this term contains a small number $1/\rho_1 \sim 10^{-5} \div 10^{-4}$ as a multiplier, the intervals of time for which the last term on the right-hand side of (3.11) may be neglected may, in effect, extend over several tens of seconds. By these approximations the Blasius equation is obtained:

$$\frac{d}{d\eta}\left(\mu'\frac{du}{d\eta}\right) + \frac{\varphi}{2}\frac{du}{d\eta} = 0 \qquad \left(\mu' = \frac{\mu}{\mu_\infty}\right) \tag{3.12}$$

The continuity equation is satisfied to the same approximation, as is easily shown by substituting into it Expressions (3.10). The boundary conditions (3.3) to (3.4) are satisfied exactly. They give

$$u(\infty) = 1, \qquad u(0) = 0, \qquad \frac{\varphi(0)}{2}(1 - c_0) = (c_\infty - c_0)\,\mu' \frac{du}{d\eta}$$

$$\frac{\varphi(0)}{2}\left\{l(T_0) + c_1(T_{-\infty})\,T_{-\infty}\,[M(n) - M(1)]\right\} = -\mu' \frac{du}{d\eta}\,[H_\infty - h_0^{(2)} + c_\infty (h_0^{(2)} - h_0^{(1)})] \tag{3.13}$$

$$c_0 = \psi(T_0)$$

Consequently, when the assumptions enumerated above are satisfied, as a result of which the right-hand side of Equation (3.11) may be neglected, the solution of the problem of sublimation of the wall is established for constant velocity of propagation of the surface of sublimation, depending only on the coordinate $x (D \sim 1/\sqrt{x})$.

To solve this problem let us introduce a new variable $\omega = \mu' du/d\eta$ and choose as independent variable a nondimensional velocity u. Then we obtain from (3.12) and (3.13)

$$\frac{d^2\omega}{du^2} + \rho\mu \frac{u}{2\omega} = 0, \qquad \omega(1) = 0, \qquad \omega'(0)(1 - c_0) = (c_0 - c_\infty)\,\omega(0) \tag{3.14}$$

$$\omega'(0)\left\{l(T_0) + c_1(T_{-\infty})\,T_{-\infty}\,[M(n) - M(1)]\right\} = \omega(0)\,[H_\infty - h_0^{(2)} + c_\infty(h_0^{(2)} - h_0^{(1)})]$$

$$c_0 = \psi(T_0)$$

The solution of this problem will be $(\rho \cdot \mu = 1)$

$$\omega(u) = u_0^{-3/2}\omega_\gamma(uu_0), \qquad \eta = \int_0^u \frac{\mu}{\omega(u)}\,du$$

$$u_0 \tan\gamma = \frac{c_0 - c_\infty}{1 - c_0}, \qquad u_0 \tan\gamma = \frac{H_\infty - h_0^{(2)} + c_\infty(h_0^{(2)} - h_0^{(1)})}{l(T_0) + c_1(T_{-\infty})\,T_{-\infty}\,[M(n) - M(1)]}, \quad c_0 = \psi(T_0) \tag{3.15}$$

where the function $u_0(\gamma)$ is calculated in [5] and is given in the table. From the last three equations c_0, T_0 and the angle γ are easily determined for every ensemble of given parameters. Knowing γ from the tables, we find u_0 and then the profile of the velocity $u(\eta)$.

The fields of concentrations and temperatures in the gas as well as in the body are defined by Equations (3.6). The coefficient of friction is found from the equation

$$c_f \sqrt{R_x} = 2\omega(0) = 2u_0^{-3/2}(\gamma) \tag{3.16}$$

The values of the function $u_0^{-3/2}(\gamma)$ are given in the table. In the

TABLE

u	$\omega_{15°}$	$\omega'_{15°}$	u	$\omega_{45°}$	$\omega'_{45°}$	u	ω_0	ω'_0
0	1.00	0.2679	0.8	1.768	0.8941	0	0	1.00
0.1	1.027	0.2655	1.0	1.942	0.8457	0.1	0.098	0.949
0.2	1.053	0.2583	1.2	2.106	0.7915	0.2	0.190	0.897
0.3	1.078	0.2466	1.4	2.258	0.7320	0.3	0.277	0.844
0.4	1.102	0.2305	1.6	2.398	0.6676	0.4	0.359	0.789
0.5	1.124	0.2103	1.8	2.525	0.5986	0.5	0.435	0.732
0.6	1.144	0.1861	2.0	2.637	0.5251	0.6	0,505	0.674
0.7	1.161	0.1580	2.2	2.735	0.4469	0.7	0.570	0.614
0.8	1.175	0.1259	2.4	3.816	0.3641	0.8	0.629	0.551
0.9	1.183	0.0899	2.6	2.880	0.2764	0.9	0.680	0.487
1.0	1.193	0.0500	2.8	2.925	0.1833	1.0	0.725	0.420
1.1	1.196	0.0060	3.0	2.952	0.0847	1.1	0.763	0.350
1.11	1.197	0.00	3.16	2.950	0.00	1.2	0.794	0.276

u	$\omega_{30°}$	$\omega'_{30°}$	u	$\omega_{60°}$	$\omega'_{60°}$	u	ω_0	ω'_0
0	1.00	0.5774	0	1.00	1.732	1.3	0.818	0.199
0.2	1.115	0.5684	0.5	1.858	1.698	1.4	0.834	0.117
0.4	1.226	0.5431	1.0	2.685	1.618	1.5	0.842	0.031
0.6	1.332	0.5043	1.5	3.466	1.517	1.55	0.843	0.00
0.8	1.429	0.4538	2.0	4.192	1.404			
1.0	1.506	0.3936	2.5	4.860	1.280			
1.2	1.581	0.3238	3.0	5.462	1.147			
1.4	1.640	0.2431	3.5	5.997	1.005			
1.6	1.683	0.1529	4.0	6.458	0.855			
1.8	1.707	0.0526	4.5	6.842	0.695			
1.88	1.708	0.00	5.0	7.143	0.525			

u	$\omega_{45°}$	$\omega'_{45°}$	u	$\omega_{60°}$	$\omega'_{60°}$
0	1.00	1.00	5.5	7.357	0.344
0.2	1.198	0.9917	6.0	7.477	0.150
0.4	1.395	0.9690	6.38	7.50	0.00
0.6	1.585	0.9357	—	—	—

$\gamma°$	$u_0(\gamma)$	$u_0^{-3/2}(\gamma)$
—90	0	∞
—75	0.25	8.51
—60	0.52	2.67
—45	0.83	1.32
—30	1.20	0.762
—15	1.60	0.494
0	2.08	0.334
15	2.73	0.222
30	3.80	0.135
45	5.75	0.0725
60	11.0	0.0274
$\gamma\to\frac{\pi}{2}$	$u_0 = 2.608^{\tan^2\gamma}$	0.00

case of large heat inflows from the gas $\tan\gamma \to \infty$. Using the asymptotic representation of the function $u_0(\gamma) = 2.608\tan^2\gamma$, we obtain for the coefficient of friction of a strongly sublimating wall the simple equation

$$c_f\sqrt{R_x} = 0.475\,(\tan\gamma)^{-3} = 1.238\,\frac{1-c_0}{c_0-c_\infty}$$

From this equation it is seen that when boiling is approached the coefficient of friction approaches zero. This effect of diminishing the coefficient of friction due to an increase of the intensity of evaporation is analogous to the effect of decreasing the coefficient of resistance of friction by blowing gas into a boundary layer [14].

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